3. Consider the following all-integer linear program:

Max $1x_1 + 1x_2$

s.t.

$$4x_1 + 6x_2 \leq 22$$
$$1x_1 + 5x_2 \leq 15$$
$$2x_1 + 1x_2 \leq 9$$
$$x_1, x_2 \geq 0 \text{ and integer}$$

a. Graph the constraints for this problem. Use dots to indicate all feasible integer solutions.
b. Solve the LP Relaxation of this problem.
c. Find the optimal integer solution.

8. Spencer Enterprises must choose among a series of new investment alternatives. The Potential investment alternatives, the net present value of the future stream of returns, the capital requirements, and the available capital funds over the next three years are summarized as follows:

Alternative	Net Present Value ($)	Capital Requirements ($) Year 1	Year 2	Year 3
Limited warehouse expansion	4,000	3,000	1,000	4,000
Extensive warehouse expansion	6,000	2,500	3,500	3,500
Test market new product	10,500	6,000	4,000	5,000
Advertising campaign	4,000	2,000	1,500	1,800
Basic research	8,000	5,000	1,000	4,000
Purchase new equipment	3,000	1,000	500	900
Capital funds available		10,500	7,000	8,750

a. Develop and solve an integer programming model for maximizing the net present value.
b. Assume that only one of the warehouse expansion projects can be implemented. Modify your model of part (a).
c. Suppose that, if test marketing of the new product is carried out, the advertising campaign also must be conducted. Modify your formulation of part (b) to reflect this new situation.

16. The Northshore Bank is working to develop an efficient work schedule for full-time and part-time tellers. The schedule must provide for efficient operation of the bank including adequate customer service, employee breaks, and so on. On Fridays the bank is open from 9:00 a.m. to 7:00 p.m. The number of tellers necessary to provide adequate customer service during each hour of operation is summarizeed here.

Time	Number Number of Tellers	Time	Number Number of Tellers
9:00 a.m.–10:00 a.m.	6	2:00 p.m.–3:00 p.m.	6
10:00 a.m.–11:00 a.m.	4	3:00 p.m.–4:00 p.m.	4
11:00 a.m.–Noon	8	4:00 p.m.–5:00 p.m.	7
Noon–1:00 p.m.	10	5:00 p.m.–6:00 p.m.	6
1:00 p.m.–2:00 p.m.	9	6:00 p.m.–7:00 p.m.	6

Each full-time employee starts on the hour and works a 4-hour shift, followed by 1 hour for lunch and then a 3-hour shift. Part-time employees work one 4-hour shift beginning on the hour. Considering salary and fringe benefits, full-time employees cost the bank $15 per hour ($105 a day), and part-time employees cost the bank $8 per hour ($32 per day).
a. Formulate an integer programming model that can be used to develop a schedule that will satisfy customer service needs at a minimum employee cost. (*Hint:* Let x_i = number of full-time

employees coming on duty at the beginning of hour i and y_i = number of part-time employees coming on duty at the beginning of hour i.)

b. Solve the LP Relaxation of your model in part (a).

c. Solve for the optimal schedule of tellers. Comment on the solution.

d. After reviewing the solution to part (c), the bank manager realized that some additional requirements must be specified. Specifically, she wants to ensure that one fulltime employee is on duty at all times and that there is a staff of at least five full-time employees. Revise your model to incorporate these additional requirements and solve for the optimal solution.

21. The Bayside Art Gallery is considering installing a video camera security system to reduce its insurance premiums. A diagram of the eight display rooms that Bayside uses for exhibitions is shown in Figure 7.13; the openings between the rooms are numbered 1 through 13. A security firm proposed that two-way cameras be installed at some room openings. Each camera has the ability to monitor the two rooms between which the camera is located. For example, if a camera were located at opening number 4, rooms 1 and 4 would be covered; if a camera were located at opening 11, rooms 7 and 8 would be covered; and so on. Management decided not to locate a camera system at the entrance to the display rooms. The objective is to provide security coverage for all eight rooms using the minimum number of two-way cameras.

a. Formulate a 0-1 integer linear programming model that will enable Bayside's management to determine the locations for the camera systems.

b. Solve the model formulated in part (a) to determine how many two-way cameras to purchase and where they should be located.

c. Suppose that management wants to provide additional security coverage for room 7. Specifically, management wants room 7 to be covered by two cameras. How would your model formulated in part (a) have to change to accommodate this policy restriction?

d. With the policy restriction specified in part (c), determine how many two-way camera systems will need to be purchased and where they will be located.

FIGURE 7.13 DIAGRAM OF DISPLAY ROOMS FOR BAYSIDE ART GALLERY